The Higgs mechanism

- Peskin & Schroeder 20
- Schwartz 28.3, 28.4

Technically, the Higgs mechanism is just the extension of SSB of global symmetries to local symmetries.

Physically, it is very different since the local part of the "gauge symmetry" is not a symmetry, but a redundancy. A_μ should be identified with $A_\mu + \partial_\mu \alpha$. This was forced by the fact that we wanted to describe massless vectors. But if now our goal is to describe massive vectors, why should we have to talk about gauge symm to begin with?

We can write down a theory of massive vectors without problem. However, it will be a non-renormalizable theory.

Given that the longitudinal polarization vectors behave as

$$\varepsilon_L^\mu \sim \frac{p^\mu}{m}$$

at high energies, the theory has a cutoff determined by the gauge boson masses themselves

$$\Lambda_{cutoff} \lesssim \frac{4\pi}{g} m$$

The physical intuition is the following: it was Lorentz invariance what forced us to "eliminate" the longitudinal polarizations in the massless case. So, for $E \gg m$, we should do something drastic with the long. modes of vector bosons.

This is where the Higgs mechanism comes in: We describe massive vectors as a spontaneously broken gauge theory.

- $U(1)$

start with the $U(1)$ gauge theory, also known as the abelian Higgs mechanism.

$$\mathcal{L} = D_\mu \phi^\dagger D^\mu \phi - V(\phi) - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

with

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

- without gauging, we have described this theory with and without SSB.
- For $-\mu^2 \to +\mu^2$, it is the theory of a massive scalar, charged, and a massless photon.
  It has a global $U(1)_Q$ & therefore preserves charge.
- In the broken theory,

$$\langle \phi \rangle = \frac{v}{\sqrt{2}} , \quad v = \frac{\mu}{\sqrt{2}} \neq 0 \longrightarrow \begin{cases} \phi = \frac{v}{\sqrt{2}} + \chi \\ \phi = \frac{v+\rho}{\sqrt{2}} e^{i\theta/v} \end{cases}$$

the polar rep. is most convenient for the following discussion. This is because under $U(1)$,

$$\rho \to \rho , \quad \theta \to \theta + \alpha(x) v$$

we get

$$D_\mu \phi = \frac{1}{\sqrt{2}} e^{i\theta/v} \partial_\mu \rho + \frac{v+\rho}{\sqrt{2}} e^{i\theta/v} i \frac{\partial_\mu \theta}{v} - i \frac{v+\rho}{\sqrt{2}} e^{i\theta/v} A_\mu$$

$$= e^{i\theta/v} \frac{1}{\sqrt{2}} \left(\partial_\mu \rho - i(v+\rho)\left(A_\mu - i \frac{\partial_\mu \theta}{v}\right)\right)$$

the $\mathcal{L}$ is

$$\mathcal{L} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(v+\rho)^2\left(A_\mu - \frac{\partial_\mu\theta}{v}\right)^2 - \lambda v^2\rho^2 - \lambda v\rho^3 - \frac{\lambda}{4}\rho^4$$
$$- \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}$$

This is gauge invariant because

$$\begin{aligned} A_\mu - \frac{\partial_\mu\theta}{v} &\to (A_\mu + \partial_\mu\alpha) - \frac{\partial_\mu(\theta + \alpha v)}{v} \\ &= A_\mu + \partial_\mu\alpha - \partial_\mu\alpha - \frac{\partial_\mu\theta}{v} \\ &= A_\mu - \frac{\partial_\mu\theta}{v} \end{aligned}$$

So the photon mass term $A_\mu A^\mu$ is gauge invariant because we accompany it with the $\partial_\mu\theta$ term.

It contains mixings $A_\mu\partial^\mu\theta$.

The simplest thing is to avoid it by fixing the gauge, which we have to do to quantize it anyway.

We choose the unitary gauge:

$$\theta = 0.$$

The field is simply given by

$$\phi(x) = \frac{v + \rho(x)}{\sqrt{2}}.$$

In general, "unitary gauge" is a gauge where we set to zero all scalars mixing with the vector fields.

The unit. gauge is the most convenient to read the spectrum of the theory.

$$\mathcal{L} = -\frac{1}{4} F^2 + \frac{1}{2}(v+\rho)^2 A_\mu A^\mu + \frac{1}{2}(\partial \rho)^2 - V(\rho)$$

The theory has one massive vector with

$$m_A^2 = e^2 v^2 = e^2 \mu^2 / \lambda$$

and one massive scalar with

$$m_\rho^2 = \sqrt{2}\, \mu^2$$

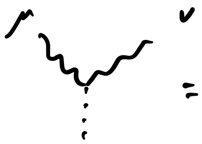
Notice that ρ is neutral, but it couples to A_μ with strength fixed in terms of m_A.

Dof unbroken : 2 real scalar + 2 pol.

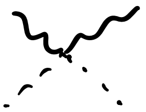
Dof broken : 1 real scalar + 3 pol.

- In unit gauge we fix $\mathcal{G}/\mathcal{H}$ but not $\mathcal{H}$. It makes it clear that goldstone are not physical. In this case the physical consequence of SSB is the mass of vectors.

Feynman rules:



$= -2 i e^2 v \, \eta_{\mu\nu} = -2 i e \, m_A \, \eta_{\mu\nu}$



$= -2 i e^2 \eta_{\mu\nu}$

$= -6 i \lambda v$

$= -6 i \lambda$

The correlation between masses and vertices, all controlled by v, e, λ, leads to an asymptotic behaviour for scattering amplitudes compatible with theory being renormalizable. In short, the apparent unitarity violation in longitudinal scattering from low energies is "cured" by the Higgs.

- We can generalize the result for a generic group $\mathcal{G}$ spont. broken to $\mathcal{H}$ by a set of scalars ϕ.

The covariant derivative of ϕ is

$$D_\mu \phi_i = \partial_\mu \phi_i - i A_\mu^A T_{ij}^A \phi_j$$

$$= - A_\mu^A \cdot i T_{ij}^A v_j + \dots \quad \leftarrow \phi = v + \chi, \text{ keep } v \text{ terms}$$

$$= - A_\mu^{\hat{a}} W_i^{\hat{a}}$$

$$\rightarrow \frac{1}{2}(D_\mu \phi)^2 = \frac{1}{2} g^2 A_\mu^{\hat{a}} A_\mu^{\hat{b}} (\vec{W}^{\hat{a}})^T \cdot (W^{\hat{b}}) + \dots$$

So the "$\hat{a}$" vectors, i.e. the ones that belong to the broken directions, have a mass matrix

$$M^2_{\hat{a}\hat{b}} = g^2 \vec{W}_{\hat{a}}^T \cdot \vec{W}_{\hat{b}}$$

Since W's are independent, the matrix has full rank and all "$\hat{a}$" vectors are massive.

In summary, a gauge theory with $\mathcal{G} \to \mathcal{H}$ has

$\dim(\mathcal{H})$ massless vectors

$\dim(\mathcal{G}/\mathcal{H})$ massive vectors

Note that there are no massless scalars. The would-be Goldstones become the longitudinal polarizations of the massive vectors.

The Higgs field are the radial ρ massive modes.

- Example: Non-abelian Higgs Model

$$\mathcal{G} = SO(3) \quad , \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$(T^A)_{ij} = \varepsilon_{Aij} \qquad \text{ex: } T_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \frac{1}{2} D_\mu \phi_i D^\mu \phi^i - \left[-\frac{m^2}{2} \phi^2 + \frac{\lambda}{8} \phi^4 \right]$$

$$; \; D_\mu \phi = \partial_\mu \phi - e A^A_\mu T^A \phi$$

Ground state: $\frac{\partial V}{\partial \phi} = 0 \longrightarrow \frac{m^2}{2} = \frac{\lambda}{4} \phi^2 \rightarrow \langle \phi \rangle^2 = v^2$
$= 2m^2 \lambda$

Vacuum manifold : S_2 . At any point,

$$SO(3) \to SO(2) = U(1).$$

Pick $\langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \to$ T_1, T_2 broken, T_3 unbroken

· General fluctuation :

$$\phi = e^{\pi_1 T_1 + \pi_2 T_2} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v+\sigma)$$

$$\equiv \underbrace{R(\pi_1, \pi_2) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{\text{spans } S_2} (v+\sigma)$$

Unitary gauge : $\pi_1 = \pi_2 = 0$. $\phi = \begin{pmatrix} 0 \\ 0 \\ v+\sigma \end{pmatrix}$

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{2}(\partial\sigma)^2 + \frac{e^2}{2}(v+\sigma)^2 \left[(A^1_\mu)^2 + (A^2_\mu)^2 \right]$$
$$- \left[-\frac{m^2}{2}(v+\sigma)^2 + \frac{\lambda}{8}(v+\sigma)^4 \right]$$

Notice that A^1, A^2 have the same mass. They form a doublet under the unbroken $U(1)_3$,

$$A^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu) = W^\pm_\mu$$

- A^3_μ remains massless, photon.
- W bosons massive, $m_W^2 = e^2 v^2$.
- Higgs massive, $m_\sigma^2 = \lambda v^2$, couples to W prop to their mass.